Chapter 9
Inferences from Two Samples

Slide 1

Overview

Slide 2

There are many important and meaningful situations in which it becomes necessary to compare **two** sets of sample data.

Inferences about Two Proportions

Slide 3

Assumptions

1. We have proportions from two **independent** simple random samples.
2. For both samples, the conditions $np \geq 5$ and $nq \geq 5$ are satisfied.

Notation for Two Proportions

Slide 4

For population 1, we let:

p_1 = population proportion

n_1 = size of the sample

x_1 = number of successes in the sample

$\hat{p}_1 = \frac{x_1}{n_1}$ (the *sample* proportion)

$\hat{q}_1 = 1 - \hat{p}_1$

The corresponding meanings are attached to p_2, n_2, x_2, $\hat{p}_2$. and $\hat{q}_2$, which come from population 2.

Pooled Estimate of p_1 and p_2

Slide 5

- The **pooled estimate of p_1 and p_2** is denoted by $\bar{p}$.

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

- $\bar{q} = 1 - \bar{p}$

Test Statistic for Two Proportions

Slide 6

For $H_0: p_1 = p_2$, $H_0: p_1 \geq p_2$, $H_0: p_1 \leq p_2$

$H_1: p_1 \neq p_2$, $H_1: p_1 < p_2$, $H_1: p_1 > p_2$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

Test Statistic for Two Proportions

Slide 7

For $H_0: p_1 = p_2$, $H_0: p_1 \ge p_2$, $H_0: p_1 \le p_2$

$H_1: p_1 \ne p_2$, $H_1: p_1 < p_2$, $H_1: p_1 > p_2$

where $p_1 - p_2 = 0$ (assumed in the null hypothesis)

$$\hat{p}_1 = \frac{x_1}{n_1} \quad \text{and} \quad \hat{p}_2 = \frac{x_2}{n_2}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{and} \quad \bar{q} = 1 - \bar{p}$$

Example: For the sample data listed in Table 8-1, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Slide 8

Table 8-1 Racial Profiling Data

	Race and Ethnicity	
	Black and Non-Hispanic	White and Non-Hispanic
Drivers stopped by police	24	147
Total number of observed drivers	200	1400
Percent Stopped by Police	**12.0%**	**10.5%**

Example: For the sample data listed in Table 8-1, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Slide 9

$n_1 = 200$

$x_1 = 24$

$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$

$n_2 = 1400$

$x_2 = 147$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$

$H_0: p_1 = p_2$, $H_1: p_1 > p_2$

$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{24 + 147}{200+1400} = 0.106875$

$\bar{q} = 1 - 0.106875 = 0.893125.$

Example: For the sample data listed in Table 8-1, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Slide 10

$n_1 = 200$

$x_1 = 24$

$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$

$n_2 = 1400$

$x_2 = 147$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$

$$z = \frac{(0.120 - 0.105) - 0}{\sqrt{\frac{(0.106875)(0.893125)}{200} + \frac{(0.106875)(0.893125)}{1400}}}$$

$z = 0.64$

Example: For the sample data listed in Table 8-1, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Slide 11

$n_1 = 200$

$x_1 = 24$

$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$

$n_2 = 1400$

$x_2 = 147$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$

$z = 0.64$

This is a right-tailed test, so the P-value is the area to the right of the test statistic $z = 0.64$. The *P*-value is 0.2611.

Because the *P*-value of 0.2611 is greater than the significance level of $\alpha = 0.05$, we fail to reject the null hypothesis.

Example: For the sample data listed in Table 8-1, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Slide 12

$n_1 = 200$

$x_1 = 24$

$\hat{p}_1 = \frac{x_1}{n_1} = \frac{24}{200} = 0.120$

$n_2 = 1400$

$x_2 = 147$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{147}{1400} = 0.105$

$z = 0.64$

Because we fail to reject the null hypothesis, we conclude that there is not sufficient evidence to support the claim that the proportion of black drivers stopped by police is greater than that for white drivers. This does *not* mean that racial profiling has been disproved. The evidence might be strong enough with more data.

Example: For the sample data listed in Table 8-1, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Slide 13

$n_1 = 200$

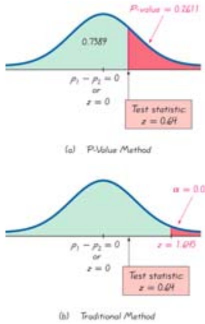
$x_1 = 24$

$\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{24}{200} = 0.120$

$n_2 = 1400$

$x_2 = 147$

$\hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{147}{1400} = 0.105$



Using TI Calculator:

Slide 14

1 EDIT CALC TESTS 1:Z-Test… 2:T-Test… 3:2-SampZTest… 4:2-SampTTest… 5:1-PropZTest… 6:2-PropZTest… 7↓ZInterval…	2 2-PropZTest x1:24 n1:200 x2:147 n2:1400 p1:≠p2 <p2 >p2 Calculate Draw
3 2-PropZTest p1>p2 z=.6422677219 p=.2603496184 p̂1=.12 p̂2=.105 ↓p̂=.106875	4 Compare these results with the example presented in the last few slides.

Confidence Interval Estimate of $p_1 - p_2$

Slide 15

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

where $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

Example: For the sample data listed in Table 8-1, find a 90% confidence interval estimate of the difference between the two population proportions.

Slide 16

$n_1 = 200$

$x_1 = 24$

$\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{24}{200} = 0.120$

$n_2 = 1400$

$x_2 = 147$

$\hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{147}{1400} = 0.105$

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$E = 1.645\sqrt{\frac{(.12)(.88)}{200} + \frac{(0.105)(0.895)}{1400}}$$

$$E = 0.400$$

Example: For the sample data listed in Table 8-1, use a 0.05 significance level to test the claim that the proportion of black drivers stopped by the police is greater than the proportion of white drivers who are stopped.

Slide 17

$n_1 = 200$

$x_1 = 24$

$\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{24}{200} = 0.120$

$n_2 = 1400$

$x_2 = 147$

$\hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{147}{1400} = 0.105$

$(0.120 - 0.105) - 0.040 < (p_1 - p_2) < (0.120 - 0.105) + 0.040$

$-0.025 < (p_1 - p_2) < 0.055$

Using TI Calculator:

Slide 18

1 EDIT CALC TESTS 6↑2-PropZTest… 7:ZInterval… 8:TInterval… 9:2-SampZInt… 0:2-SampTInt… A:1-PropZInt… B:2-PropZInt…	2 2-PropZInt x1:24 n1:200 x2:147 n2:1400 C-Level:.9 Calculate
3 2-PropZInt (-.0251,.05513) p̂1=.12 p̂2=.105 n1=200 n2=1400	4 Compare these results with the example presented in the last few slides.

Definitions

Slide 19

Two Samples: Independent

The sample values selected from one population are not related or somehow paired with the sample values selected from the other population.

If the values in one sample are related to the values in the other sample, the samples are <u>dependent</u>. Such samples are often referred to as <u>matched pairs</u> or <u>paired samples</u>.

Assumptions

Slide 20

1. **The two samples are *independent*.**
2. **Both samples are *simple random samples.***
3. **Either or both of these conditions are satisfied: The two sample sizes are both *large* (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions.**

Hypothesis Tests

Slide 21

Test Statistic for Two Means:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Hypothesis Tests

Slide 22

Test Statistic for Two Means:

Degrees of freedom: In this book we use this estimate: df = smaller of $n_1 - 1$ and $n_2 - 1$.

***P*-value:** Refer to Table A-3. Use the procedure summarized in Figure 7-6.

Critical values: Refer to Table A-3.

McGwire Versus Bonds

Slide 23

Data Set 30 in Appendix B includes the distances of the home runs hit in record-setting seasons by Mark McGwire and Barry Bonds. Sample statistics are shown. Use a 0.05 significance level to test the claim that the distances come from populations with different means.

	McGwire	Bonds
n	70	73
$\bar{x}$	418.5	403.7
s	45.5	30.6

McGwire Versus Bonds

Slide 24

Claim: $\mu_1 \neq \mu_2$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

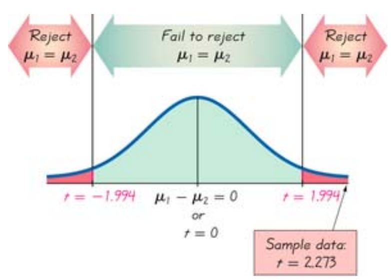
$\alpha = 0.05$

$n_1 - 1 = 69$

$n_2 - 1 = 72$

$df = 69$

$t_{.025} = 1.994$



McGwire Versus Bonds

Slide 25

Test Statistic for Two Means:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

McGwire Versus Bonds

Slide 26

Test Statistic for Two Means:

$$t = \frac{(418.5 - 403.7) - 0}{\sqrt{\dfrac{45.5^2}{70} + \dfrac{30.6^2}{73}}}$$

$$= 2.273$$

McGwire Versus Bonds

Slide 27

Claim: $\mu_1 \neq \mu_2$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$\alpha = 0.05$

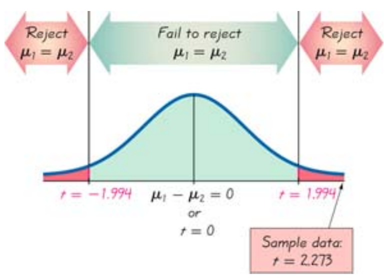


Figure 8-2

McGwire Versus Bonds

Slide 28

Claim: $\mu_1 \neq \mu_2$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$\alpha = 0.05$

There is significant evidence to support the claim that there is a difference between the mean home run distances of Mark McGwire and Barry Bonds.

Reject Null

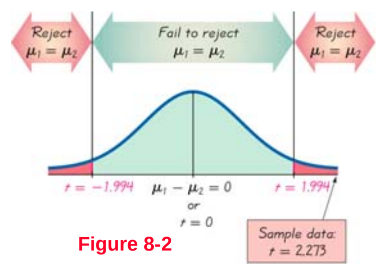
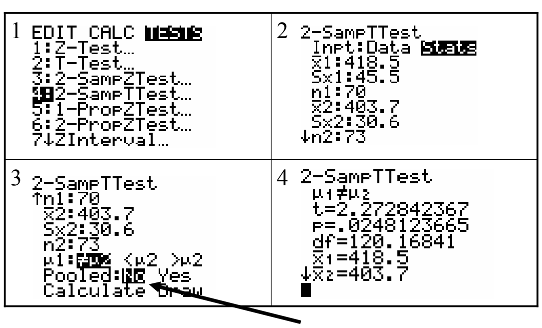


Figure 8-2

Using TI Calculator:

Slide 29



Our textbook recommends to use No Pooled option at this time.

Confidence Intervals

Slide 30

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where $E = t_{\alpha/2} \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

McGwire Versus Bonds

Slide 31

Using the sample data given in the preceding example, construct a 95%confidence interval estimate of the difference between the mean home run distances of Mark McGwire and Barry Bonds.

$$E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$E = 1.994 \sqrt{\frac{45.5^2}{70} + \frac{30.6^2}{73}}$$

$$E = 13.0$$

McGwire Versus Bonds

Slide 32

Using the sample data given in the preceding example, construct a 95%confidence interval estimate of the difference between the mean home run distances of Mark McGwire and Barry Bonds.

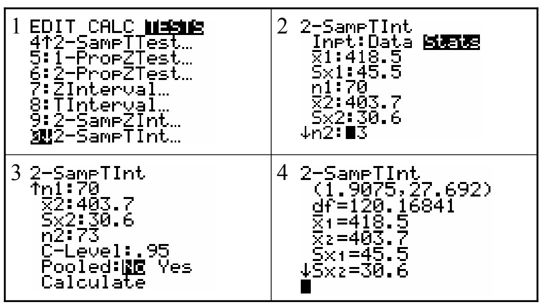
$$(418.5 - 403.7) - 13.0 < (\mu_1 - \mu_2) < (418.5 - 403.7) + 13.0$$

$$1.8 < (\mu_1 - \mu_2) < 27.8$$

We are 95% confident that the limits of 1.8 ft and 27.8 ft actually do contain the difference between the two population means.

Using TI Calculator:

Slide 33



Assumptions

Slide 34

1. **The sample data consist of matched pairs.**
2. **The samples are simple random samples.**
3. **Either or both of these conditions is satisfied: The number of matched pairs of sample data is (n > 30) or the pairs of values have differences that are from a population having a distribution that is approximately normal.**

Notation for Matched Pairs

Slide 35

μ_d = **mean value of the differences d for the population of paired data**

$\bar{d}$ = **mean value of the differences d for the paired sample data (equal to the mean of the $x - y$ values)**

s_d = **standard deviation of the differences d for the paired sample data**

n = **number of pairs of data.**

Test Statistic for Matched Pairs of Sample Data

Slide 36

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

where degrees of freedom = $n - 1$

P-values and Critical Values

Slide 37

Use Table A-3 (t-distribution).

Confidence Intervals

Slide 38

$$\bar{d} - E < \mu_d < \bar{d} + E$$

where $E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}$

degrees of freedom = n –1

Are Forecast Temperatures Accurate?

Slide 39

Table 8-2 Actual and Forecast Temperature

Actual low	1	−5	−5	23	9
Low forecast five days earlier	16	16	20	22	15
Difference d = actual − predicted	−15	−21	−25	1	−6

$\bar{d}$ = –13.2, s = 10.7, n = 5

$t_{\alpha/2}$ = 2.776 (found from Table A-3 with 4 degrees of freedom and 0.05 in two tails)

Are Forecast Temperatures Accurate?

Slide 40

H_0: $\mu_d = 0$

H_1: $\mu_d \neq 0$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{-13.2 - 0}{\frac{10.7}{\sqrt{5}}} = -2.759$$

Because the test statistic does not fall in the critical region, we fail to reject the null hypothesis.

Are Forecast Temperatures Accurate?

Slide 41

H_0: $\mu_d = 0$

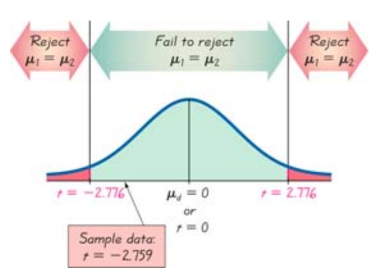
H_1: $\mu_d \neq 0$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{-13.2 - 0}{\frac{10.7}{\sqrt{5}}} = -2.759$$

The sample data in Table 8-2 do not provide sufficient evidence to support the claim that actual and five-day forecast low temperatures are different.

Are Forecast Temperatures Accurate?

Slide 42



Are Forecast Temperatures Accurate?

Slide 43

Using the same sample matched pairs in Table 8-2, construct a 95% confidence interval estimate of μ_d, which is the mean of the differences between actual low temperatures and five-day forecasts.

Are Forecast Temperatures Accurate?

Slide 44

$$E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

$$E = (2.776)\left(\frac{10.7}{\sqrt{5}}\right)$$

$$= 13.3$$

Are Forecast Temperatures Accurate?

Slide 45

$$\bar{d} - E < \mu_d < \bar{d} + E$$

$$-13.2 - 13.3 < \mu_d < -13.2 + 13.3$$

$$-26.5 < \mu_d < 0.1$$

Are Forecast Temperatures Accurate?

Slide 46

In the long run, 95% of such samples will lead to confidence intervals that actually do contain the true population mean of the differences.

Using TI Calculator:

Slide 47

1 Enter actual low into L1, predicted values into L2, and the difference into L3 as explained in class.	2 L1 L2 L3 3 1 16 -5 16 -5 20 23 22 9 15 ------ ------ ------ L3 =L1−L2
3 L1 L2 L3 3 1 16 -15 -5 16 -21 -5 20 -25 23 22 1 9 15 -6 ------ ------ L3(6) =	4 T-Test Inpt:Data Stats μ0:0 List:L3 Freq:1 μ:≠μ0 <μ0 >μ0 Calculate Draw

Using TI Calculator:

Slide 48

5 T-Test μ≠0 t=-2.762014037 p=.0507446727 x̄=-13.2 Sx=10.68644001 n=5	6 TInterval Inpt:Data Stats List:L3 Freq:1 C-Level:.95 Calculate
7 TInterval (-26.47,.06897) x̄=-13.2 Sx=10.68644001 n=5	8 Compare these results with the example presented in the last few slides.

Measures of Variation

Slide 49

s = standard deviation of sample

σ = standard deviation of population

s^2 = variance of sample

σ^2 = variance of population

Assumptions

Slide 50

1. The two populations are *independent* of each other.
2. The two populations are each normally distributed.

Notation for Hypothesis Tests with Two Variances

Slide 51

s_1^2 = *larger* of the two sample variances

n_1 = size of the sample with the *larger* variance

σ_1^2 = variance of the population from which the sample with the *larger* variance was drawn

The symbols s_2^2, n_2, and σ_2^2 are used for the other sample and population.

Test Statistic for Hypothesis Tests with Two Variances

Slide 52

$$F = \frac{s_1^2}{s_2^2}$$

Critical Values: Using Table A-5, we obtain critical F values that are determined by the following three values:

1. The significance level α.
2. Numerator degrees of freedom (df_1) = $n_1 - 1$
3. Denominator degrees of freedom (df_2) = $n_2 - 1$

Slide 53

- All one-tailed tests will be right-tailed.
- All two-tailed tests will need only the critical value to the right.
- When degrees of freedom is not listed exactly, use the critical values on either side as an interval. Use interpolation only if the test statistic falls within the interval.

Slide 54

If the two populations do have equal variances, then $F = \frac{s_1^2}{s_2^2}$ will be close to 1 because s_1^2 and s_2^2 are close in value.

If the two populations have radically different variances, then F will be a large number.

Remember, the larger sample variance will be s_1 .

Slide 55

Consequently, a value of F near 1 will be evidence in favor of the conclusion that $\sigma_1^2 = \sigma_2^2$.

But a large value of F will be evidence against the conclusion of equality of the population variances.

Coke Versus Pepsi

Slide 56

Data Set 17 in Appendix B includes the weights (in pounds) of samples of regular Coke and regular Pepsi. Sample statistics are shown. Use the 0.05 significance level to test the claim that the weights of regular Coke and the weights of regular Pepsi have the same standard deviation.

	Regular Coke	Regular Pepsi
n	36	36
$\bar{x}$	0.81682	0.82410
s	0.007507	0.005701

Coke Versus Pepsi

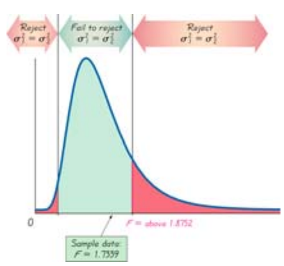
Slide 57

Claim: $\sigma_1^2 = \sigma_2^2$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.05$



$$\text{Value of } F = \frac{s_1^2}{s_2^2} = \frac{0.007507^2}{0.005701^2} = 1.7339$$

Coke Versus Pepsi

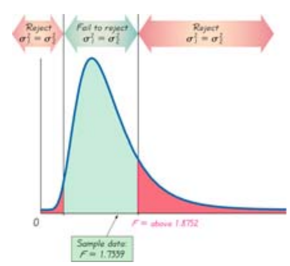
Slide 58

Claim: $\sigma_1^2 = \sigma_2^2$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

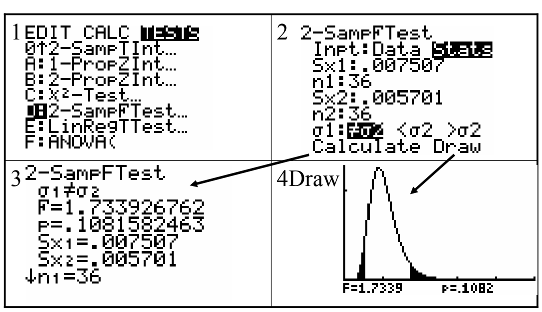
$\alpha = 0.05$



There is not sufficient evidence to warrant rejection of the claim that the two variances are equal.

Using TI Calculator:

Slide 59



Finding Lower Critical F Values

Slide 60

1) Use $\mathbf{F_R}$ indicates the critical value for right tail and $\mathbf{F_L}$ indicates the critical value for the left tail.
2) Interchange the degrees of freedom.
3) $\mathbf{F_L}$ is the reciprocal of the **F** value found in the table.

For example: n_1=7, n_2=10, α=0.05

Answer: $\mathbf{F_L}$**=0.1810,** $\mathbf{F_R}$**=4.3197**